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Low-temperature behavior of magnetic metamaterial elements

Kamil Boratay Alici¹ and Ekmel Ozbay

Nanotechnology Research Center, Department of Physics,
Department of Electrical and Electronics Engineering, Bilkent University,
Bilkent, 06800 Ankara, Turkey
E-mail: bora@fen.bilkent.edu.tr

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Abstract. Periodically arranged metallic resonators can produce a negative permeability medium. However, the resonant response weakens at extreme regimes under certain conditions, which is the major problem of obtaining a negative index in the visible regime. We report that by decreasing the operation temperature, the metal conductivity can be increased, enhanced negative permeability can be obtained and the operation range of the negative permeability media, and thereby the negative index media, can be extended. We doubled the resonant strength of a typical resonator operating at microwave frequencies by decreasing its temperature to 150 K. The results are promising for the demonstration of negative index media in the visible regime.

Metrology is one of the key elements of nanotechnology. Conventional lenses are limited by diffraction and, therefore, an imaging resolution that is less than the wavelength (λ) of incident light cannot be obtained. Currently, to view objects as small as 10 nm in size, we use electron beam-based microscopy. Recently, a novel mechanism for imaging was introduced: a flat lens that has an effective negative index of refraction [1]. Subwavelength imaging in turn became possible and an image of sources placed at a distance $\lambda/6$ apart was reported [2]. If we could synthesize a negative index lens of $\lambda/20$ resolution, we would be able to view objects 20 and 10 nm in size with blue ($\lambda = 400$ nm) or UV ($\lambda = 200$ nm) light sources, respectively. Negative index materials are a subset of metamaterials whose electromagnetic properties can be controlled at will, in a narrow frequency band, by specifying the ingredients of the metamaterial's unit cell.

¹ Author to whom any correspondence should be addressed.

Every crystalline material occurring in nature is composed of periodically arranged elements. When the wavelength of incident electromagnetic radiation is much larger than the material's unit cell dimensions, we can use an effective medium approach to understand the material's response to the incident radiation [3]. Two unique complex quantities, permittivity $\varepsilon(\omega)$ and permeability $\mu(\omega)$, are assigned to the material medium in order to identify its complete electromagnetic response. In 1999, Pendry *et al* proposed a feasible metamaterial design to extend the naturally available range of ε and μ to include negative values [4]. The idea was experimentally verified by the Smith group in the X-band (8–12 GHz) by using a composite medium of wire mesh and split ring resonators that provided negative ε and μ , respectively [5]. The verification of metamaterial properties at megahertz [6], millimeter [7], terahertz [8] and optical [9]–[11] frequency regimes was rather difficult. In order to obtain metamaterials operating at higher frequencies, the physical size of the metamaterial unit cells has to be smaller. In the process of unit cell size scaling, it was observed that due to the losses in metallic components, the magnetic resonance strength saturates [12]. In addition, the fabrication of circularly shaped metallic resonators becomes more complicated as the operation frequency increases [10, 11]. These obstacles led researchers to come up with novel designs called planar metamaterials [9]–[11], [13, 14]. Cut-wire pairs, which provide magnetic resonance, had a smaller total capacitance compared with the same size split ring resonator. Therefore, cut-wire-based metamaterials can operate at higher frequencies, which was proposed as a solution to the saturation and fabrication problems [14]. The design and fabrication solutions that have been provided in this direction, especially to demonstrate negative index metamaterials at optical frequencies, have attracted much attention [8, 12]. One critique should be emphasized at this point: in order to be able to obtain an effective medium, the electrical size of the composing elements should be at least an order of magnitude smaller than the operation wavelength. The planar metamaterial approach fails to provide this condition: the electrical size of the cut-wire pairs was close to $\lambda/2$ and not much smaller than the operation wavelength. In the present paper, we would like to discuss another possible solution to the weakening response of metamaterial elements. It is well known that the properties of metallic features are strongly dependent on the environment temperature [15]; surprisingly, there has been no direct observation of the low-temperature behavior of metamaterials that are composed of metallic elements. We propose to decrease the operation temperature of metamaterial elements, which could be a candidate solution to the weak response of negative permeability media.

We noticed very recently that while trying to obtain deep subwavelength magnetic metamaterial elements at microwave frequencies, due to the ohmic losses of the constituting metallic elements with thin features, their magnetic resonance saturates [16]. The saturation was directly related to the resistance and electrical size of the resonators. Thereby, we expected that the reduction of conductive losses would increase the resonance strength and thereby larger negative permeability values could be obtained. In order to understand the underlying physics, we selected a specific type of element, as shown in figure 1, and investigated the theoretical formalism that has been developed in the literature [4, 17]. The effective permeability of a medium composed of periodically arranged subwavelength resonators can be written as

$$\mu_{\text{eff}} = 1 - \frac{F}{1 - \omega_0^2/\omega^2 + i1/Q}.$$

Here, F is the fractional volume constant, ω_0 the resonance frequency, ω the frequency of the incident electromagnetic wave and Q the resonator quality factor. The Q -factor related to

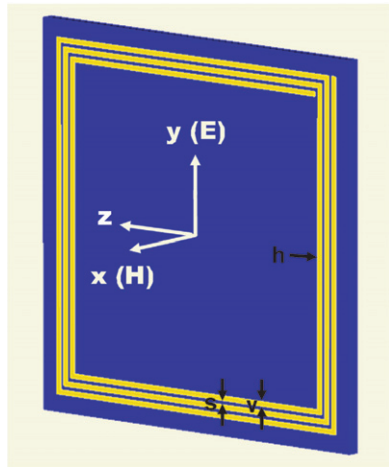


Figure 1. SR geometry: side length, $l = 8.0$ mm, width of the strips, $v = 100$ μm , separation between the strips, $s = 100$ μm , number of turns, $N = 3$, thickness of the substrate, $t = 254$ μm , and deposited copper thickness, $h = 9$ μm .

the metal losses of the spiral resonators (SR) is: $Q_c = \omega_0 L / R_c$. The Q -factor related to the dielectric losses (Q_d) [18] was found to be negligible and we used $Q = Q_c$. L is the resonator inductance, R_c is the resistance associated with the metallic losses: $R_c = L / \sigma_c v h \mu_0$. Here, σ_c is the conductivity, h the deposited metal thickness and $\sigma_c = n e^2 \tau / m$. The free electron density is represented by n , e is the magnitude of the electronic charge, τ the mean free time and m the electronic mass [15]. The temperature-dependent quantity is the mean free time τ . For copper, $\tau = 0.27$ fs at 0°C and $\tau = 2.1$ fs at liquid nitrogen temperature [19]. At lower temperatures, the mean free path, conductivity and Q -factor increase, in turn leading to larger negative effective permeability values.

In figure 1 the electrically small element that was used as the unit cell of a μ -negative medium is shown. Metallic features are obtained by etching the deposited copper with a thickness of $h = 9$ μm coated on the RT/duroid substrate with a thickness of $t = 254$ μm . The listed relative permittivity and dissipation factor at 10 GHz are $\epsilon = 2.0$ and $\tan \delta = 0.0009$. However, we used a slightly different relative permittivity, which was $\epsilon = 1.54$. The parameters of the SR elements are as follows: width of the strips, $v = 100$ μm ; separation between the strips, $s = 100$ μm ; side length, $l = 8.0$ mm; and number of turns, $N = 3$. While defining the electrical size of an element, we consider the minimum sphere that encloses it. We find the free space wavelength (λ_0) at the resonance frequency and identify the element's electrical size by dividing the sphere diameter by the free space wavelength: $u = 2a / \lambda_0$.

The effective permittivity of the medium of densely packed resonators was found to be [17]

$$\epsilon_{\text{eff}} = \epsilon_0 \epsilon \left(1 + \frac{p_y l}{p_x p_z} \frac{K(\sqrt{1-k^2})}{K(\sqrt{k})} \right).$$

Here, K denotes the complete elliptic integral of the first kind while $k = s / (s + 2v)$. The periods in the x -, y - and z -directions are denoted by p_x , p_y and p_z , respectively. We assumed a closely packed medium with $p_x = 4.1$ mm and $p_y = p_z = 8.2$ mm. This formula is the low-frequency permittivity of the medium due to the inter-cell coupling of the resonators.

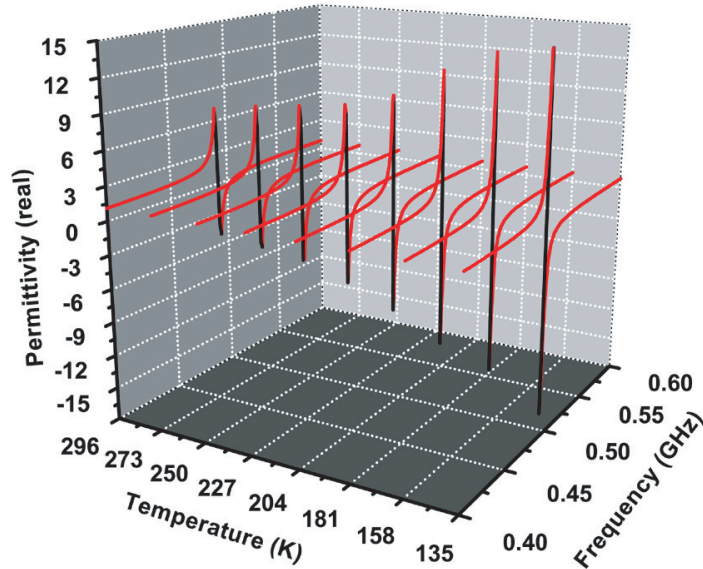


Figure 2. Theoretically calculated real part of the effective permeability of an SR based on a closely packed metamaterial medium. Data are shown for the selected temperature values.

Recently, we observed that as we attempt to decrease the electrical size of the SRs by introducing more turns, the resonant strength obtained from the transmission measurements reduced and saturated. When we changed the resonator geometry by introducing splits on the rolled long metal thin strips, we obtained higher resonance strengths. These studies led us to investigate the effect of another parameter, temperature, which would increase the resonance strength while keeping the element geometry and electrical size the same.

The temperature-dependent resistivity of copper is given by the Mathissen rule as [19] $\rho_{\text{tot}} = \rho_{\text{res}} + \rho(T)$. Here ρ_{res} denotes the residual resistivity that is independent of temperature and $\rho(T)$ is the resistivity for the ideal lattice of the material: $\rho(T) = \rho(0)r(T)/r(0)$. Here r is the reduced resistance: $r(T) = 1.056(T/\theta)F(T/\theta)$, T/θ is the reduced temperature and θ is the Debye temperature, $\theta = 347$ K for copper. The values of $F(T/\theta)$ in the temperature range of interest were taken from resistivity tables given in [20]. By using the tabulated experimental data [19], we calculated $\rho_{\text{res}} = 0.0151 \mu\Omega \text{ cm}$ and $r(0) = 0.7604$. By using the specific formulae [18], we calculated the capacitance, inductance and Q -factor of our resonator and derived the effective permeability and effective permittivity values for the temperature range of interest. Note that instead of the formula given for the calculation of effective substrate permittivity, $\epsilon_r^{\text{sub}} = 1 + (2/\pi) \arctan(t/2\pi(v+s))(\epsilon - 1)$ [18], we used ϵ directly. In figure 2, the enhancement of negative permeability for lower temperatures can be clearly seen.

As we obtained the permeability and permittivity values, the transmission and reflection coefficients can be calculated by using the formalism developed for a homogeneous slab. After calculation of the S_{21} amplitude, we comment on the resonance strength. First, we found the effective index and impedance values by using $n_{\text{eff}} = \sqrt{\epsilon_{\text{eff}}\mu_{\text{eff}}}$, $z_{\text{eff}} = \sqrt{\mu_{\text{eff}}/\epsilon_{\text{eff}}}$. Then the transmission amplitude was found by

$$S_{21} = \left[\cos(n_{\text{eff}}kd) - \frac{i}{2} \frac{z^2 + 1}{z} \sin(n_{\text{eff}}kd) \right]^{-1}.$$

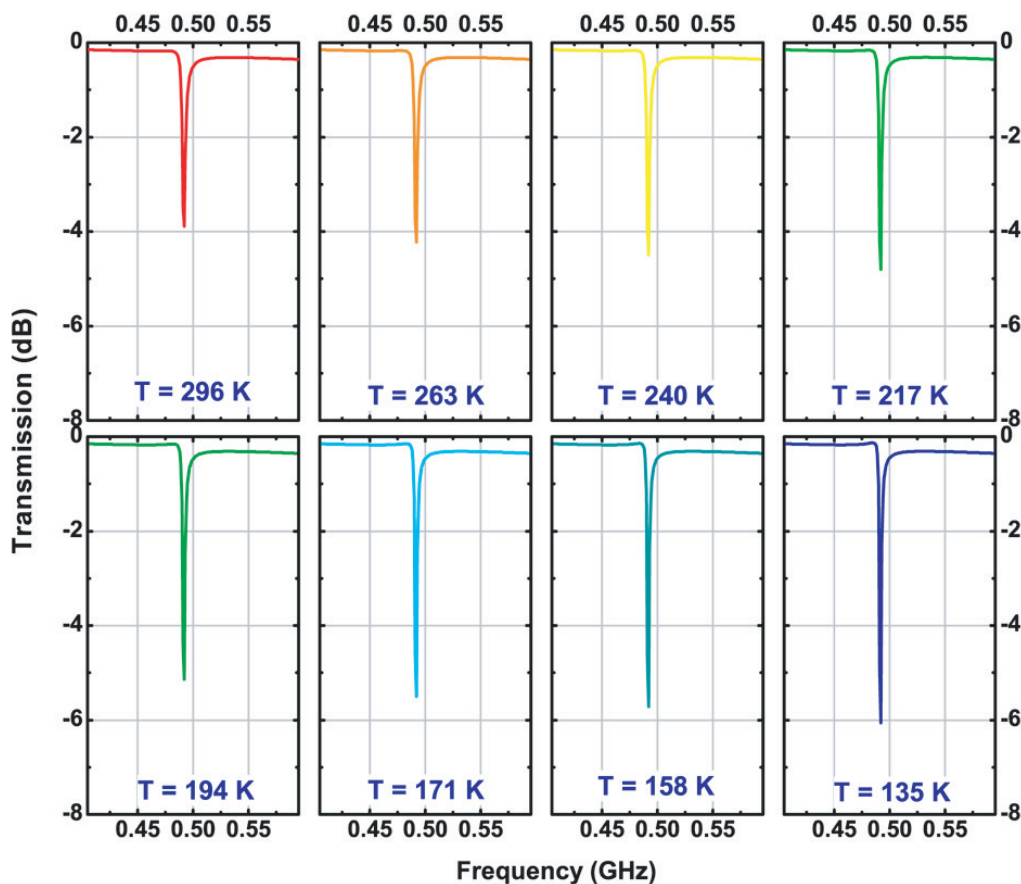


Figure 3. Theoretically calculated transmission amplitude data as a function of frequency. The results are plotted with 23 K temperature steps.

Here, k is the wavevector, $k = \omega/c$, and d is the slab thickness in the propagation direction: $d = p_z$ in our case. In figure 3, we show the temperature-dependent S_{21} data. As we decreased the temperature, the resonance strength increased in accordance with enhanced negative permeability.

To verify these theoretical results, we constructed an experimental setup and investigated the temperature dependence of the resonance quality of SR elements as shown in figure 4. The setup consists of two commonly grounded microwave probe antennas, liquid nitrogen, a platinum resistance and computer-controlled instruments: an Agilent N5230A Vector Network Analyzer and an Agilent AG34401 high-performance digital multimeter. The probe antennas operate in the reactive near-field regime. During the transmission measurement, we placed a single element between the antennas wherein we obtained the strongest response. The transmission spectra of free space, i.e. before the element was inserted, was used as calibration data. Instead of through calibration, we used this method in order to clearly see the changes of the resonant response during the experiment. When everything was in place, we poured the liquid nitrogen into the can up to the level of the antennas. The temperature of the sample was read from the multimeter with the aid of a platinum sensor, which was placed at the same level as the sample. As liquid nitrogen evaporates, the temperature of the sample increases and eventually reaches room temperature. The computer-aided automatic setup records the transmission response during the evaporation process. The temperature-dependent free space

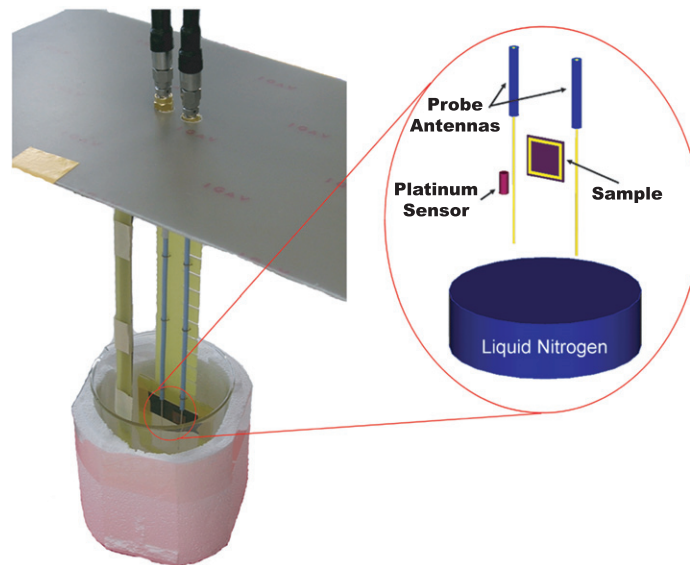


Figure 4. Experimental setup and schematic.

calibration data were obtained similarly. We repeated the experiment for different samples and obtained their temperature-dependent calibrated transmission responses.

In figure 5 we show the transmission response of the SR at different temperatures. Starting from the laboratory temperature, the transmission data are shown down to 135 K in steps of 23 K. In this setup, since we kept the antennas at a higher level than the liquid nitrogen as shown in figure 4, the lowest temperature value we obtained was around 125 K higher than the liquid nitrogen temperature of 77 K. We saw that the decreasing temperature did not have any effect on the resonance frequency. On the other hand, due to the increased conductivity, the resonance strength significantly increased. We also tested SRs with $N = 4$ and 20. For the $N = 4$ case, the effect was minor, and for the $N = 20$ case we did not see any change. To be able to increase the resonance strength of samples with a longer metal length, we had to reduce the temperature further, which was only possible with a cryogenic experimental setup. In figure 5, we show the results of the obtained dip and peak values at the resonance frequency as the temperature changed. For this particle, even if its electrical size was kept the same, the resonant strength doubled by decreasing its temperature to 150 K. Doubling the resonance strength means larger negative permeability values, or in other words negative permeability values for saturated elements at room temperature.

In principle, we can obtain a negative permeability medium by using electrically small split ring resonators under a low-temperature environment at optical frequencies. The mechanism works by decreasing the losses and not by inhibiting the saturation of resonance frequency due to the kinetic energy of electrons in the metal. By using subwavelength elements together with wire mesh at a low temperature, we can also obtain a negative index flat lens and convey the current metrology to a higher level.

In summary, we report the low-temperature response of negative permeability medium elements. Just as the theoretical calculations predicted, we observed higher resonance strength and larger negative permeability values as we decreased the temperature. Increasing the metal conductivity via this method helps to solve the problem of the weakening negative permeability

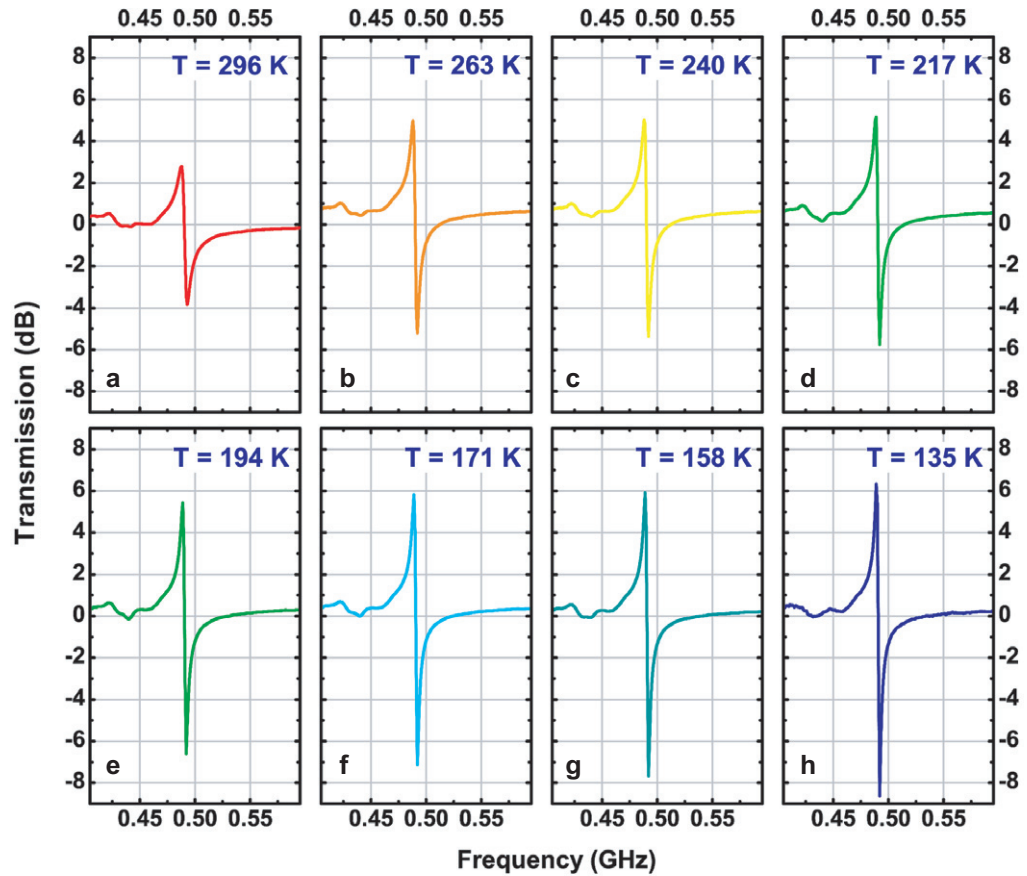


Figure 5. Calibrated experimental transmission amplitude data as a function of frequency. The results are plotted with 23 K temperature steps.

response of metamaterials and extends the limits of negative index media. This method can retain the electrical size of the constituting elements and thereby the subwavelength resolution ability of negative index metamaterials.

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