Experimental observation of cavity formation in composite metamaterials

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Abstract: In this paper, we investigated one of the promising applications of left-handed metamaterials: composite metamaterial based cavities. Four different cavity structures operating in the microwave regime were constructed, and we observed cavity modes on the transmission spectrum with different quality factors. The effective permittivity and permeability of the CMM structure and cavity structure were calculated by use of a retrieval procedure. Subsequently, in taking full advantage of the effective medium theory, we modeled CMM based cavities as one dimensional Fabry-Perot resonators with a subwavelength cavity at the center. We calculated the transmission from the Fabry-Perot resonator model using the one-dimensional transfer matrix method, which is in good agreement with the measured result. Finally, we investigated the Fabry-Perot resonance phase condition for a CMM based cavity, in which the condition was satisfied at the cavity frequency. Therefore, our results show that it is possible to treat metamaterial based cavities as one-dimensional Fabry-Perot resonators with a subwavelength cavity.

OCIS codes: (160.3918) Materials: Metamaterials; (260.2065) Physical optics: Effective medium theory.

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1. Introduction

Materials with simultaneously negative permittivity (ε) and negative permeability (μ) and, therefore, with a negative index of refraction, are known as the secret garden of electromagnetism, since there is no naturally occurring negative index material. Although, in 1968, Veselago [1] proposed that there is not any restriction to obtaining these materials by the laws of electromagnetism, this proposition was not demonstrated for four decades. The structure consisting of periodically arranged metallic wires is shown to exhibit a plasma cut-off frequency, below which the material is opaque [2]. Therefore, it is possible to obtain negative ε, using periodically arranged metallic thin wires. However, the main problem is to construct a medium with negative permeability because of the lack of a magnetic charge. In 1999, Pendry et al. suggested split-ring resonator (SRR) structures and demonstrated that such structures strongly respond to an incident magnetic field resulting in negative permeability [3]. The possibility of the materials with negative permittivity (ε) and negative permeability (μ) opened the door of the secret garden and subsequently attracted much attention in the scientific community.

This idea was experimentally investigated by the construction of a composite metamaterial (CMM) or left-handed metamaterial consisting of two components (thin wires and SRR structures) that have negative permittivity and negative permeability simultaneously over a certain frequency range [4]. Since then, various studies that employed different structure designs and applications extended this investigation. Negative refraction [5-9], subwavelength focusing of light [10-12], reverse Doppler shift [13], or cloaking [14] are some of these applications.

In the present paper, we investigated one of the promising applications of left-handed metamaterials: CMM based cavities. We constructed four different cavity structures operating in the microwave regime. The cavity modes were observed in the transmission spectrum. We

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calculated the effective ε and μ of the CMM structure and cavity structure by use of the retrieval procedure. Consequently, we modeled CMM based cavities as one-dimensional (1D) Fabry-Perot resonators (FPRs) with a subwavelength cavity at the center. The transmission was calculated from the FPR model using the 1D transfer matrix method. Finally, we investigated the resonant condition at the cavity frequency for a CMM based cavity structure.

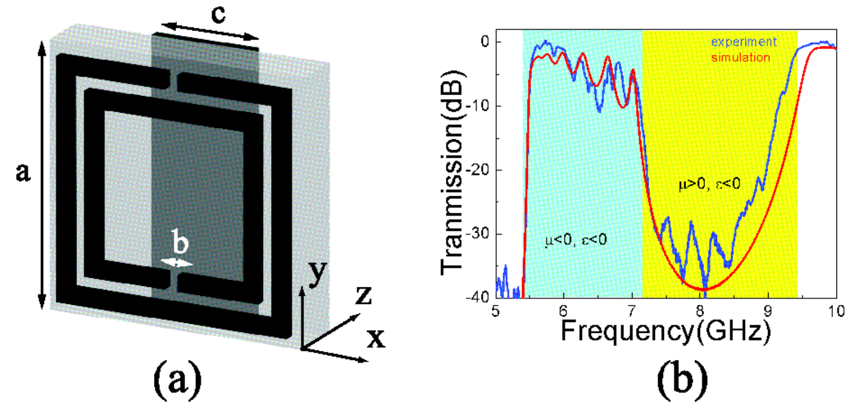

Fig. 1. (a). The unit cell of the CMM structure: a=4.95 mm, b= 0.25 mm, and c=1.6 mm. The wire stripes were printed on the back of Teflon (ε=2.17) substrates, and the square SRR were printed on the front faces. The thickness of the metal (copper) was 0.05 mm. (b). The measurement (blue line), which is in agreement with the calculation (red line), demonstrates that this CMM structure has a left-handed transmission peak from 5.5-7.0 GHz. The simulation was performed by the commercial software program CST Microwave Studio®.

2. Composite metamaterial structure

The metamaterial medium that we used in this study was composed of a 1D periodic arrangement of wire stripes and square SRR structures. The wire stripes were printed on the back of Teflon (ε=2.17) substrates, and the square SRR were printed on the front faces. The thickness of the Teflon substrate was 1mm. The unit cell of the CMM structure is shown in Fig. 1(a). The thickness of the metal (copper) was 0.05 mm. The width of the wire stripes was 1.6 mm. The lattice constant along the x direction was 4.95 mm and along the x direction (propagation direction) it was 3 mm. There were 40 layers along the y and z directions and 8 layers along the propagation direction. The E-field was in the y direction. The experimental setup consisted of an HP 8510C network analyzer and two standard gain horn antennae in order to measure the transmission amplitude.

The measured transmission demonstrates that this CMM structure has a left-handed transmission peak from 5.5-7.0 GHz [Fig. 1(b)]. We calculated the transmission using the commercial software program CST Microwave Studio®. The calculated transmission is in good agreement with the measured result. On the other hand, this CMM structure possesses a reflective medium from 7.0-9.5 GHz, or in other words, it has negative ε in this frequency region.

3. Composite metamaterial cavity structures

We examined the properties of the perfectly periodic system. Therefore, we would like to discuss the LHM systems wherein the translational symmetry is broken by a defect. It is possible to obtain a cavity mode in the forbidden transmission region of the CMM structure by introducing a defect in the structure. We designed four different defect structures: (1) D1: closed ring on the front side of the board and cut wire on the back side, (2) D2: closed ring only on the front side, (3) D3: cut wire only on the back side and, (4) D4: cut wire on both sides of the Teflon board (Fig. 2). These defect structures are introduced in the center of the CMM structure.

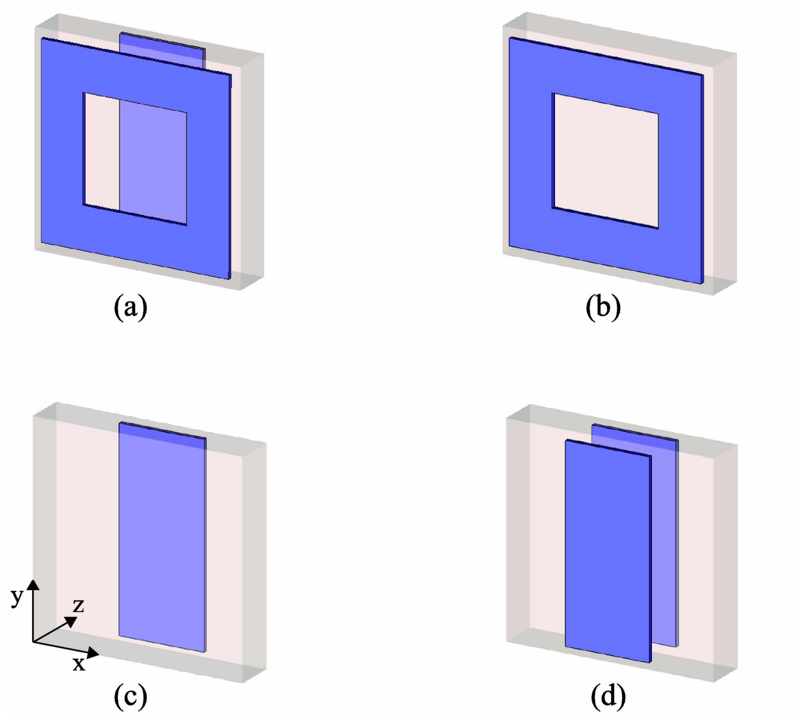

Fig. 2. Four different defect structures used in this study: (a) D1: closed ring on the front side of the board and cut wire on the back side, (b) D2: closed ring only on the front side, (c) D3: cut wire only on the back side and, (d) D4: cut wire on both sides of the Teflon board. These defect structures were introduced in the center of the CMM structure. Therefore, there are four layers of a CMM structure in the forward and backward of the defect structures in the propagation direction.

Therefore, there are four layers of the CMM structure in the forward and backward of the defect structures in the propagation direction. We observed cavity modes in the transmission spectrum of the CMM-based cavities (Fig. 3). The experiments are in good agreement with those calculations that were performed by using CST Microwave Studio®. All of these defect structures allow for a cavity mode in the transmission spectrum with different Q-factors (quality factor, defined as the center frequency, divided by the full width at half maximum) (Table 1). Therefore, it is possible to design different CMM based cavity structures operating at different frequencies with different Q-factors.

Table 1: Quality factors of the CMM based cavity structures

	D1	D2	D3	D4
Frequency(GHz)	8.1	8.2	8.5	7.9
Q-factor	220	215	235	275

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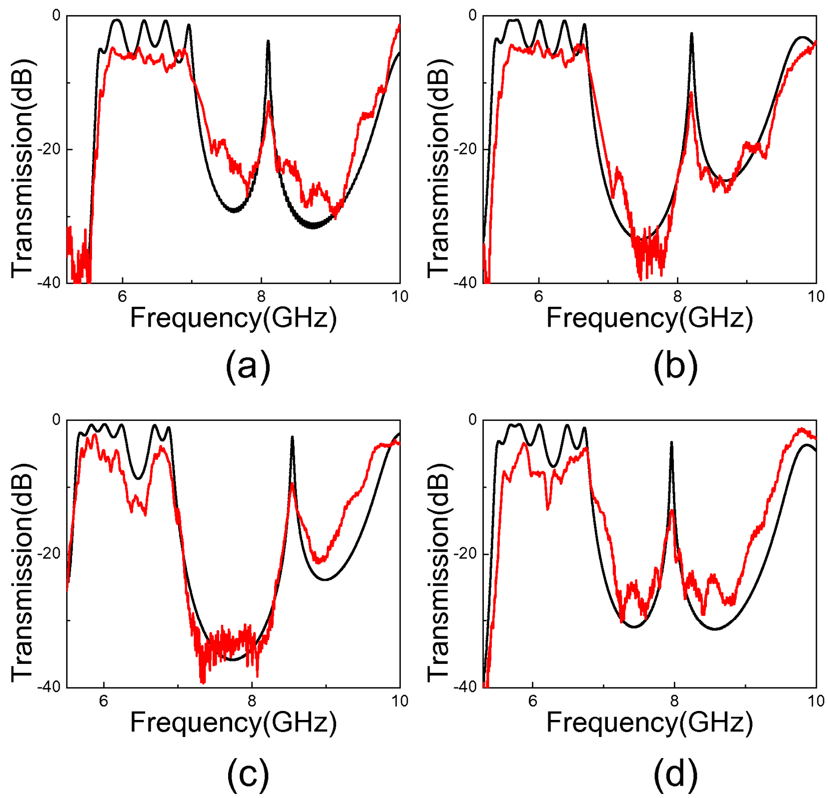

Fig. 3. We observed cavity modes in the transmission spectrum of the CMM-based cavities (D1, D2, D3, and D4). The experiments (red line) are in good agreement with the CST Microwave calculations (black line). It is possible to design different CMM based cavity structures operating at different frequencies with different Q-factors.

The reflection of the CMM structure is very high in the negative ε (and positive μ) frequency range [15] and, therefore, the CMM structures on both sides of the cavity behave like frequency-specific mirrors. The reflections of the EM waves between these two mirrors interfere constructively and destructively, giving rise to a standing wave pattern between the mirror surfaces, just like in Fabry-Perot resonances.

4. One-dimensional Fabry-Perot resonator model

One of the important properties of the metamaterial is to have a unit cell much smaller than the operating wavelength. Therefore, the metamaterials can be treated as a homogeneous medium with an effective refractive index [16-19]. The retrieval procedure is widely used to calculate the effective parameters of the metamaterials. In this method, the real and imaginary parts of the refractive index, wave impedance and, therefore, the real and imaginary parts of the ε and μ are retrieved from the amplitude and the phase information of the transmission and reflection.

We calculated the effective ε and μ of the CMM structure and defect structure by use of a retrieval procedure. The retrieval procedure is widely used to calculate the effective parameters of the metamaterials [16-22]. In this method, the real and imaginary parts of the refractive index, wave impedance and, therefore, the real and imaginary parts of the ε and μ are retrieved from the amplitude, and the phase information of the transmission and reflection.

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The details of the retrieval procedure that were used in this study are outlined in Refs.21. This particular method has the advantage of identifying the correct branch of the effective ε and μ. The ambiguity in the determination of the correct branch is resolved by the use of an analytic continuation procedure. There was one layer of the structure along the propagation direction in this calculation. We employed periodic boundary conditions along directions other than the propagation direction. Therefore, the simulation setup coincides with a slab of material that consists of a single layer. The effective ε and μ were derived from the transmission and reflection coefficients. The calculated parameters show that the CMM structure possesses effective $\varepsilon<0$, $\mu<0$ from 5.5-7.0 GHz and $\varepsilon<0$, $\mu>0$ from 7.0-9.5 GHz [Fig. 4(a)]. However, the D3 structure has $\varepsilon>0$, $\mu>0$ in this frequency range [Fig. 4(b)].

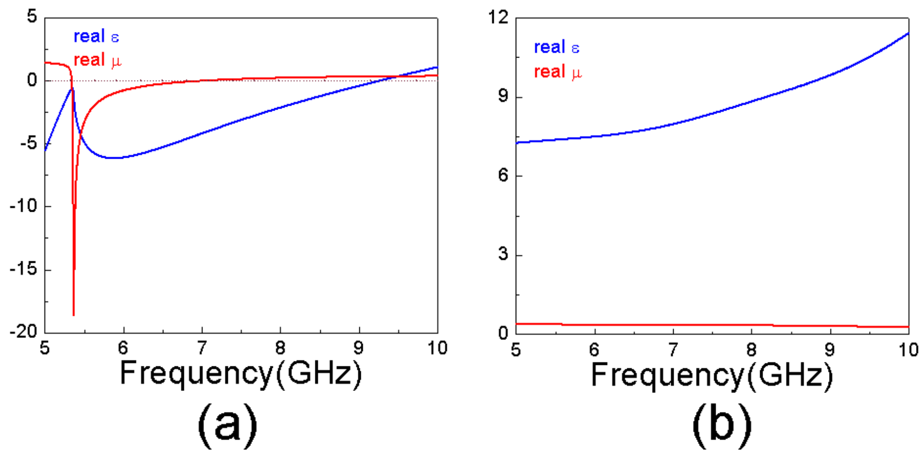

Fig. 4. (a). The calculated effective ε and μ of the CMM structure by use of a retrieval procedure show that the CMM structure possess effective $\varepsilon<0$, $\mu<0$ from 5.5-7.0 GHz and $\varepsilon<0$, $\mu>0$ from 7.0-9.5 GHz. (b) However, a D3 structure has $\varepsilon>0$, $\mu>0$ in these frequency ranges.

Consequently, the information from the retrieval procedure and transmission spectrum of the CMM cavity structures led us to model CMM based cavities as 1D FPRs with the subwavelength cavity region at the center [Fig. 5(a)]. By taking full advantage of this FPR model, we calculated the transmission from the CMM based cavity (D3) structure using the 1D transfer matrix method (TMM) [23, 24]. In the TMM calculations, we used effective ε and μ determined by use of the retrieval procedure and Fresnel coefficients (reflection coefficient and transmission coefficient) for an interface when one of the two media is left-handed [25]. The calculated transmissions for the model using 1D TMM and for the actual system using CST Microwave Studio are in good agreement [Fig. 5(b)]. Therefore, it is also in agreement with the measurements.

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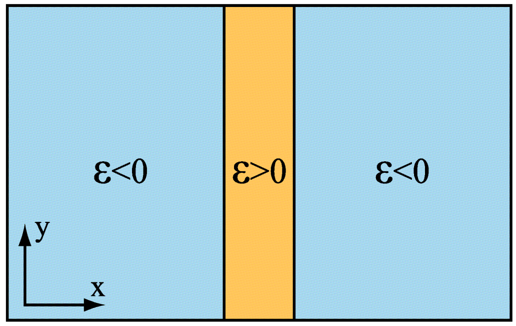
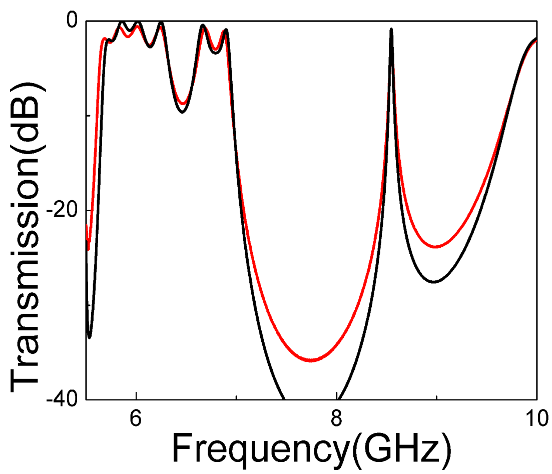


(a)



(b)

Fig. 5. (a). The CMM based cavities can be treated as 1D FPRs with a subwavelength cavity region at the center. (b). The calculated transmission from the model of the CMM based cavity (D3) structure using the 1D transfer matrix method (black line) and CST Microwave Studio (red line) are in good agreement.

Finally, in order to demonstrate the validity of the FPR model, we investigated the resonant condition at the cavity frequency for a CMM based cavity structure. We considered each of the four layers of the CMM structure as the mirrors of the FPR with the reflection coefficient $re^{-i\phi}$, where ϕ is the reflection-phase factor in radians. In a FPR, the circulating field E_c can be written as a function of the incident field E_i as:

$$E_c = \frac{t}{1 - r^2 e^{-2i(\beta L + \phi)}} E_i \tag{1}$$

where β is the propagation constant of cavity structure, L is the length of the cavity, and t is the transmission coefficient of the mirror. This equation is valid also for our model. In our model, the mirrors are 4 layer CMM structure and ϕ is the reflection-phase from this structure. The resonant condition is satisfied when $\phi_T = \beta L + \phi = m\pi$ (m=0, ±1, ±2…), where the enhancement of the circulating fields results in a high transmission at the resonant frequencies [26, 27]. For our model, this resonance condition can be viewed as cavity resonance, where a narrow transmission window within the band gap region is observed. We calculated the reflection-phase (ϕ) for 4 layer CMM using CST MWS and total phase by adding the additional phase component from the cavity structure. The calculated total phase (ϕ_T) around the cavity resonance shows that the resonance condition is satisfied at the cavity mode frequency for a CMM based cavity structure, just like in FPRs (Fig. 6).

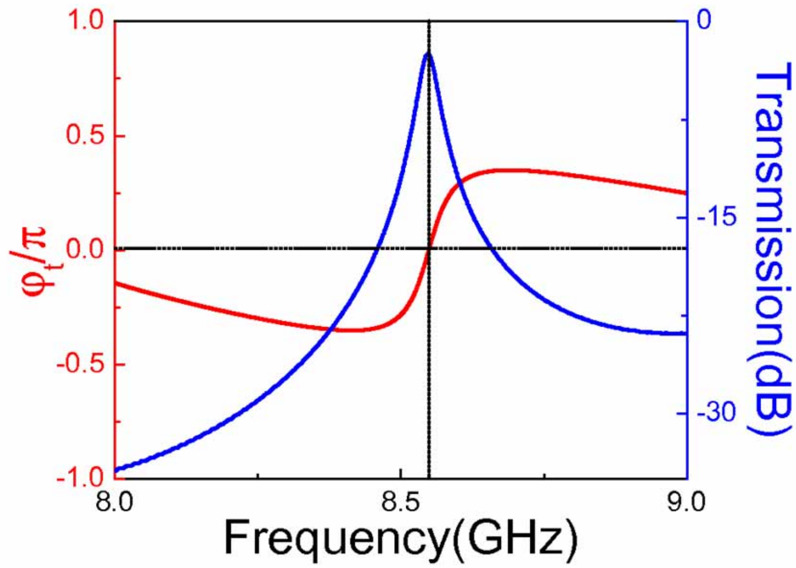


Fig. 6. The calculated total phase (ϕ_T) around the cavity resonance shows that the resonance condition is satisfied at the resonance frequency for a CMM based cavity (D3) structure, just like in FPRs.

5. Conclusion

In conclusion, we investigated cavity formation in metamaterials by using four different CMM based cavity structures. We observed the cavity mode in the transmission spectrum with different Q-factors. Then, we calculated the effective ε and μ of the CMM structure and defect (D3) structure by the use of a retrieval procedure. Eventually, taking full advantage of the effective medium theory, we modeled the CMM based cavities as 1D FPRs with a subwavelength cavity at the center. The calculated transmission from the 1D FPR model using 1D TMM is in good agreement with the measured result. Finally, we demonstrated that the Fabry-Perot resonance phase condition is satisfied at the resonance frequency of the CMM based cavity. Therefore, our results show that it is possible to treat metamaterial based cavities as 1D FPRs with a subwavelength cavity. The proposed cavity structures can be extended to optical frequencies and can be used in several applications, such as in the strong localization of the field in a subwavelength region.

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